LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2025**

PST 2501 – ESTIMATION THEORY

Date: 24-04-2025 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION –A

Answer any FOUR of the following **4 x 10 = 40 Marks**

1. If δ_1^* is a UMVUE and δ_2^* is bounded UMVUE then show that $\delta_1^* . \delta_2^*$ is also UMVUE.
2. State and prove Rao-Blackwell and Lehmann-Scheffe theorems.
3. Let $X_1, X_2, \ldots X_n$ be a random sample of size n from $U(0,\theta)$, $\theta > 0$. Obtain a sufficient statistic for θ.
4. Show with an example each that MLE is not sufficient and unique.
5. Let $X_1, X_2, \ldots X_n$ be a random sample of size n from $f(x;\theta) = \exp\{-(x-\theta)\}$, $x \geq \theta$, zero elsewhere. Find UMVUE of θ.
6. Let the following table

x	0	1	2	3	4	5
f	6	10	14	13	6	1

represent a summary of a sample of size 50 from a binomial distribution having n = 5. Find the MLE of $P(X \geq 3)$.

7. (a) Establish the invariance property of CAN estimator.
   (b) Let $X_1, X_2, \ldots X_n$ be a random sample from $N(\theta,1)$, $\theta \in R$. Find a consistent estimator for θ. (5+5)
8. Explain Jackknife and Bootstrap resampling methods.

SECTION -B

Answer any THREE of the following. **3 x 20 = 60 Marks**

9. State and prove the necessary and sufficient condition for an estimator is UMVUE using the uncorrelated approach.
10. State and prove Cramer-Rao inequality after stating the regularity conditions.
11. Let $X_1, X_2, \ldots X_n$ be a random sample of size n from $N(\mu,\sigma^2)$,where μ *and* σ^2 are unknown . Find UMVUE of (i) μ (ii) σ^2 and (iii) μ^2 / σ^2 . (5+5+10)
12. Let $X_1, X_2, \ldots X_n$ be a random sample from $N(\theta,1)$, $\theta \in R$. Show that sample mean and variance are independent after proving Basu's theorem.
13. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu,\sigma^2)$, $\mu \in R$ & $\sigma^2 > 0$.

    Find M.L.E. of (i) μ when σ^2 is known (ii) σ^2 when μ is known and (iii) $\theta = (\mu,\sigma^2)$ when μ and σ^2 are unknown.

14. Let $X \sim U(0,\theta)$, $\theta > 0$. Assume that the prior distribution of θ is $h(\theta) = \theta \exp(-\theta)$, $\theta > 0$.

    Find Bayes' estimator of θ if the loss function is (i) squared error and (ii) absolute error.

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